Journal Club
21 June 2015
Sebastiano Ferraris

Comparing algorithms for diffeomorphic registration: Stationary LDDMM and Diffeomorphic Demons

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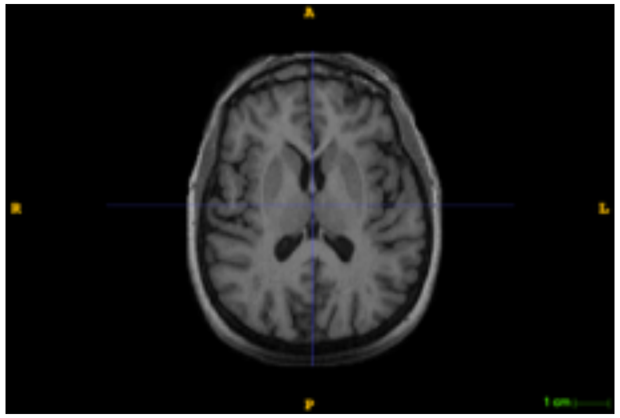
2nd MICCAI Workshop on Mathematical Foundations of Computational Anatomy, Oct 2008, New-York, United States. pp.24-35, 2008

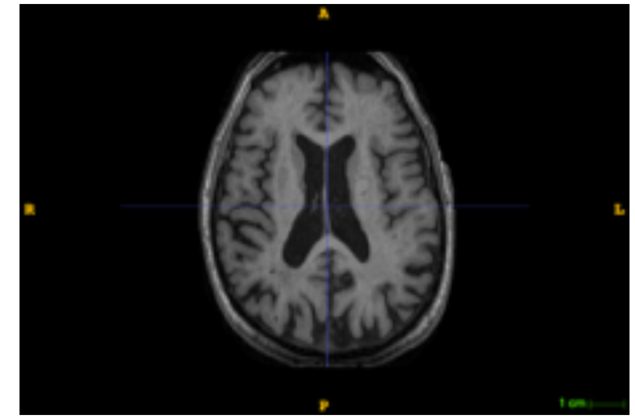
Image registration problem

Using diffeomorphisms



$$F : \mathbb{R}^2 \supset \Omega_F \longrightarrow \mathbb{R}$$

φ



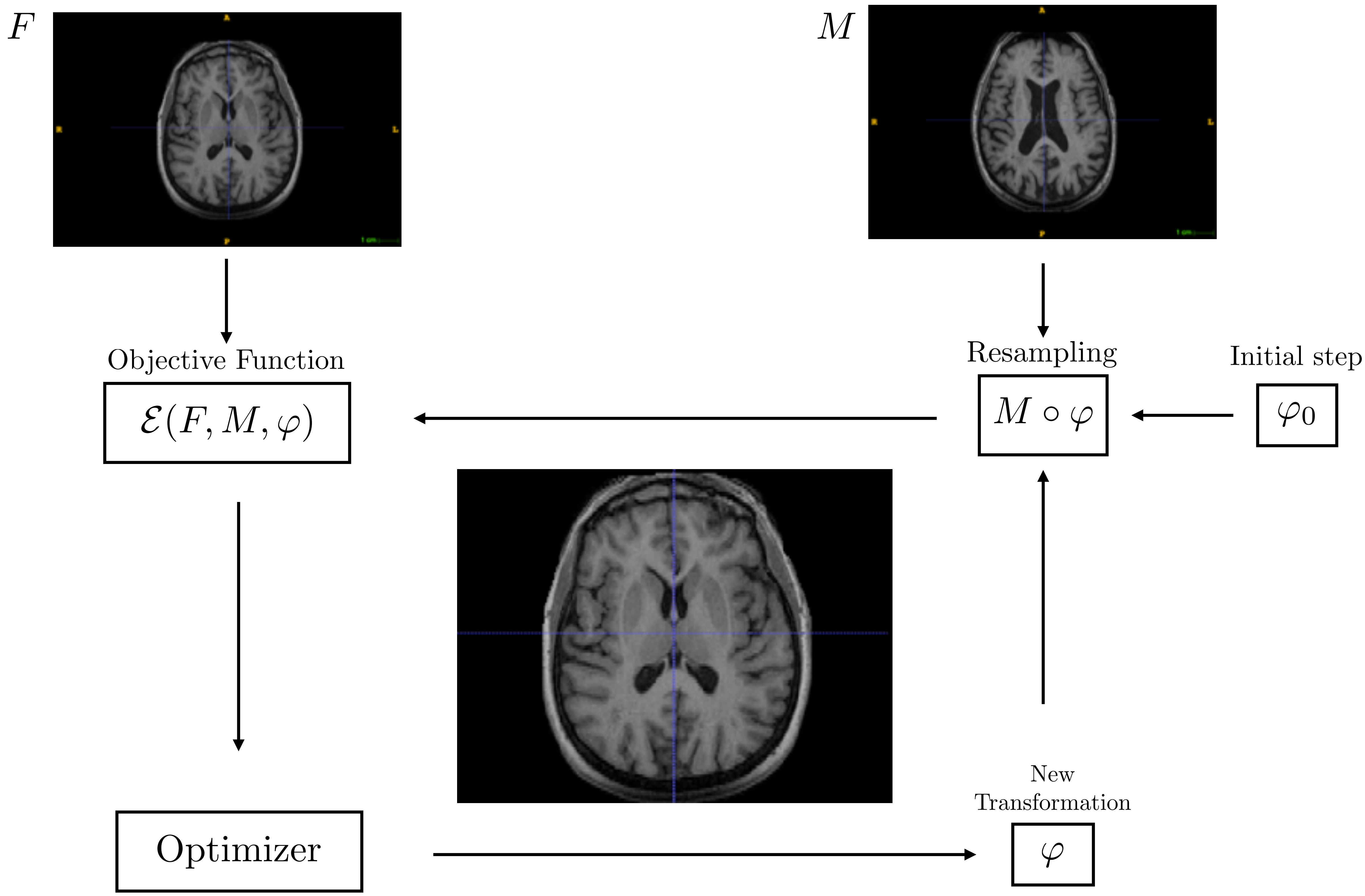
$$M : \mathbb{R}^2 \supset \Omega_M \longrightarrow \mathbb{R}$$

$$\begin{tikzcd}
\Omega_F \arrow[r, "\varphi"] \arrow[d, "F"] & \Omega_M \arrow[d, "M"] \\
\mathbb{R} \arrow[r, dashed, "\mathrm{f}"] & \mathbb{R}
\end{tikzcd}$$

Groups of diffeomorphisms

$$\boxed{\forall \mathbf{x} \in \Omega_F \qquad F(\mathbf{x}) \simeq M(\varphi(\mathbf{x})) \qquad \varphi \in \text{Set of transformations}}$$

Image registration framework



Some events:

1981

Elastic Body

Chaim Broit. *Optimal Registration of Deformed Images*.

1996

Large deformation kinematics

Christensen et. al. *Deformable templates using large deformation kinematics*.

1998

Flow of diffeomorphisms for image matching

Dupuis et al. *Variational problems on flows of diffeomorphisms for image matching*.

Demons

Thirion. *Image matching as a diffusion process: an analogy with maxwell's demons*.

2005

LDDMM

Beg, et al. *Computing large deformation metric mappings via geodesic flows of diffeomorphisms*.

2006

Log-euclidean framework and SVF

Arsigny et al. *Statistics on diffeomorphisms in a log-euclidean framework*.

2007

Stationary LDDMM

Hernandez et al. *Registration of anatomical images using geodesic paths of diffeomorphisms*.

DARTEL

Ashburner. *A fast diffeomorphic image registration algorithm*.

Log-Demons

Vercauteren et al. *Non-parametric diffeomorphic image registration with the demons algorithm*.

2008

Symmetric Log-Demons

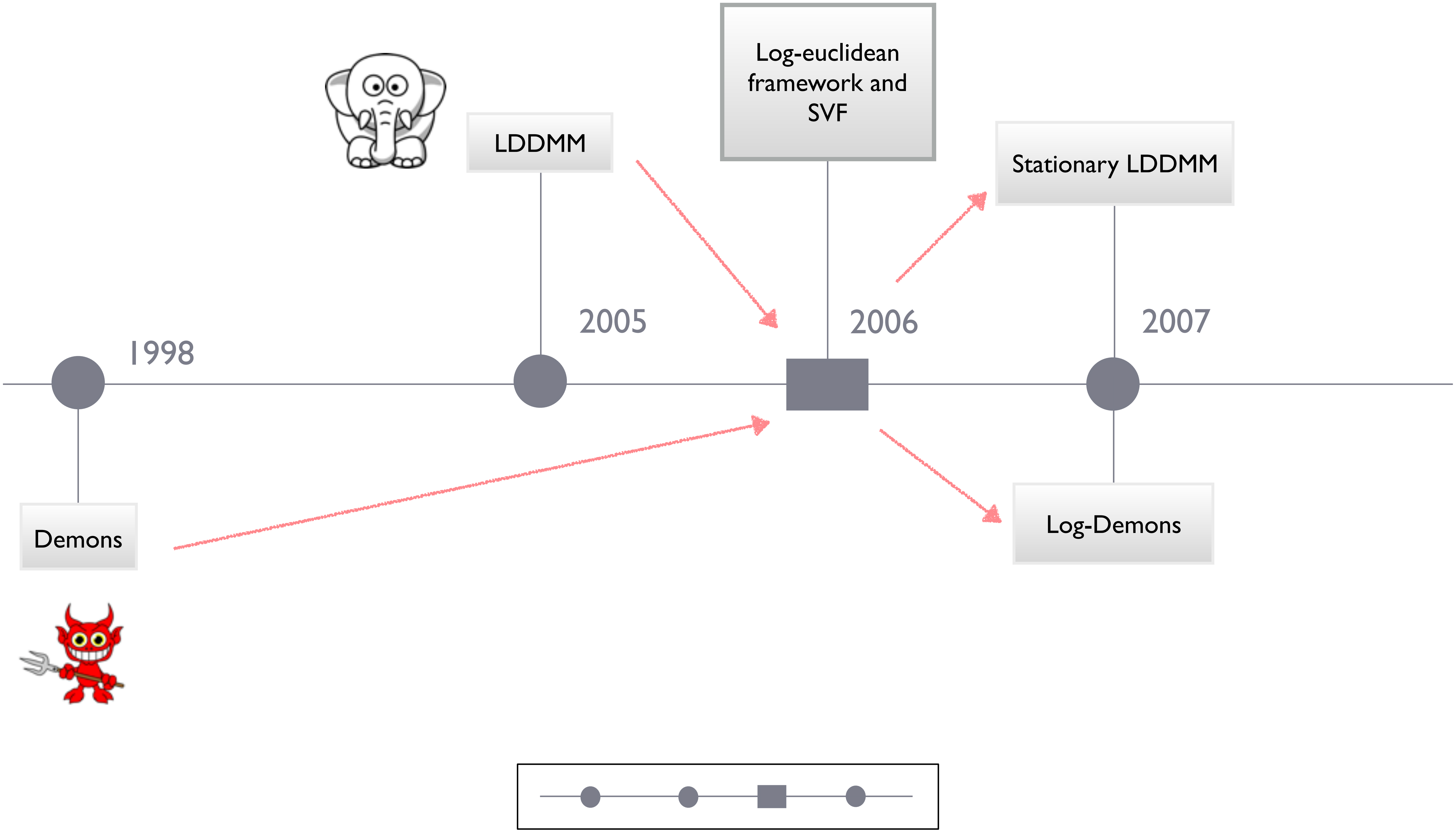
Vercauteren et al. *Symmetric log-domain diffeomorphic registration*.

2013

LCC-Demons

Lorenzi et al. *LCC-Demons: a robust and accurate symmetric diffeomorphic registration algorithm*.

Some events relevant for the paper presented today:



Demons

The original Demons consider a single vector at each voxel, ‘almost’ (up to a Gaussian filter) independent within each others.

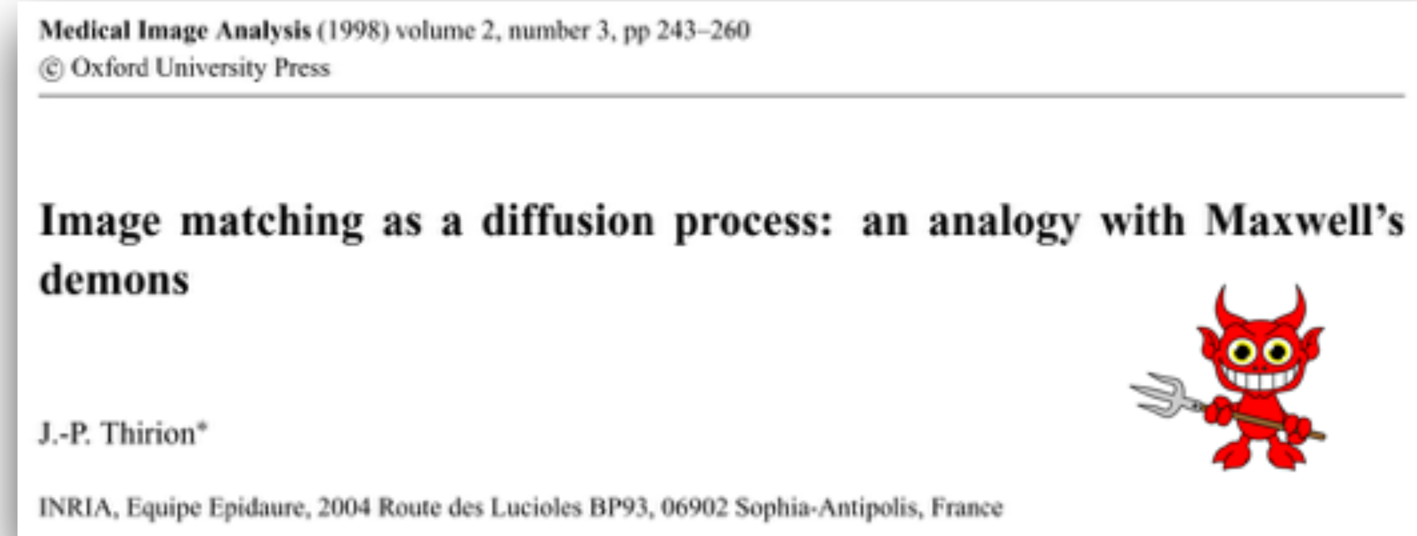
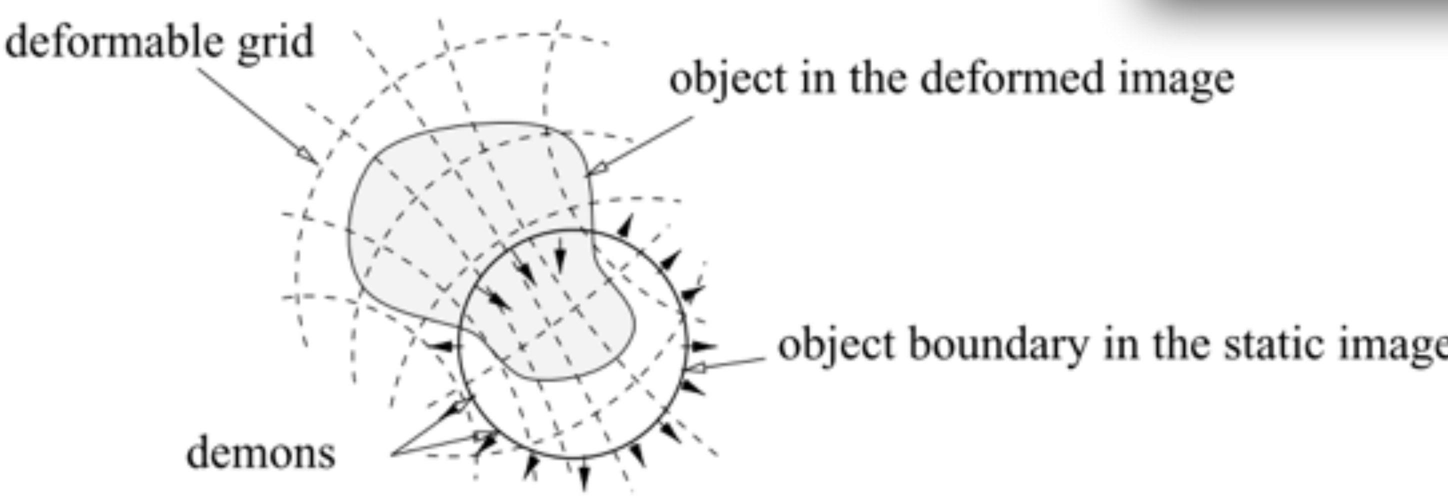
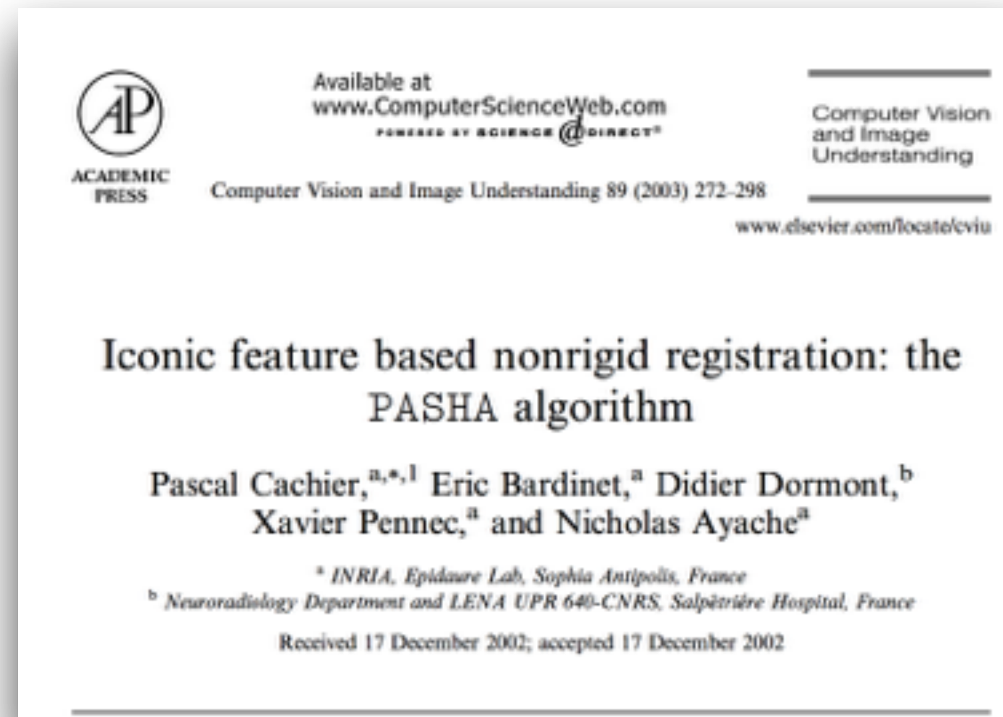

Medical Image Analysis (1998) volume 2, number 3, pp 243–260

Image matching as a diffusion process: an analogy with Maxwell’s demons

J.-P. Thirion*

INRIA, Equipe Epidaure, 2004 Route des Lucioles BP93, 06902 Sophia-Antipolis, France

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Available at www.ComputerScienceWeb.com
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Computer Vision and Image Understanding 89 (2003) 272–298
Computer Vision and Image Understanding
www.elsevier.com/locate/cviu

Iconic feature based nonrigid registration: the PASHA algorithm

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Received 17 December 2002; accepted 17 December 2002

PASHA algorithm, instead of independent vectors, consider a vector field and an optimisation function that takes it as argument.

$$\mathcal{E}(F, M, \mathbf{v}) = \frac{1}{\sigma_s}\mathrm{Sim}(F, M, \mathbf{v}) + \mathrm{Reg}(\mathbf{v})$$

LDDMM

$$\mathcal{E}(F, M, \mathbf{v}) = \frac{1}{\sigma_s}\mathrm{Sim}(F, M, \mathbf{v}) + \mathrm{Reg}(\mathbf{v})$$

$$\mathrm{Sim}(F, M, \mathbf{v}) = ||M \circ \varphi - F||^2_{L^2}$$

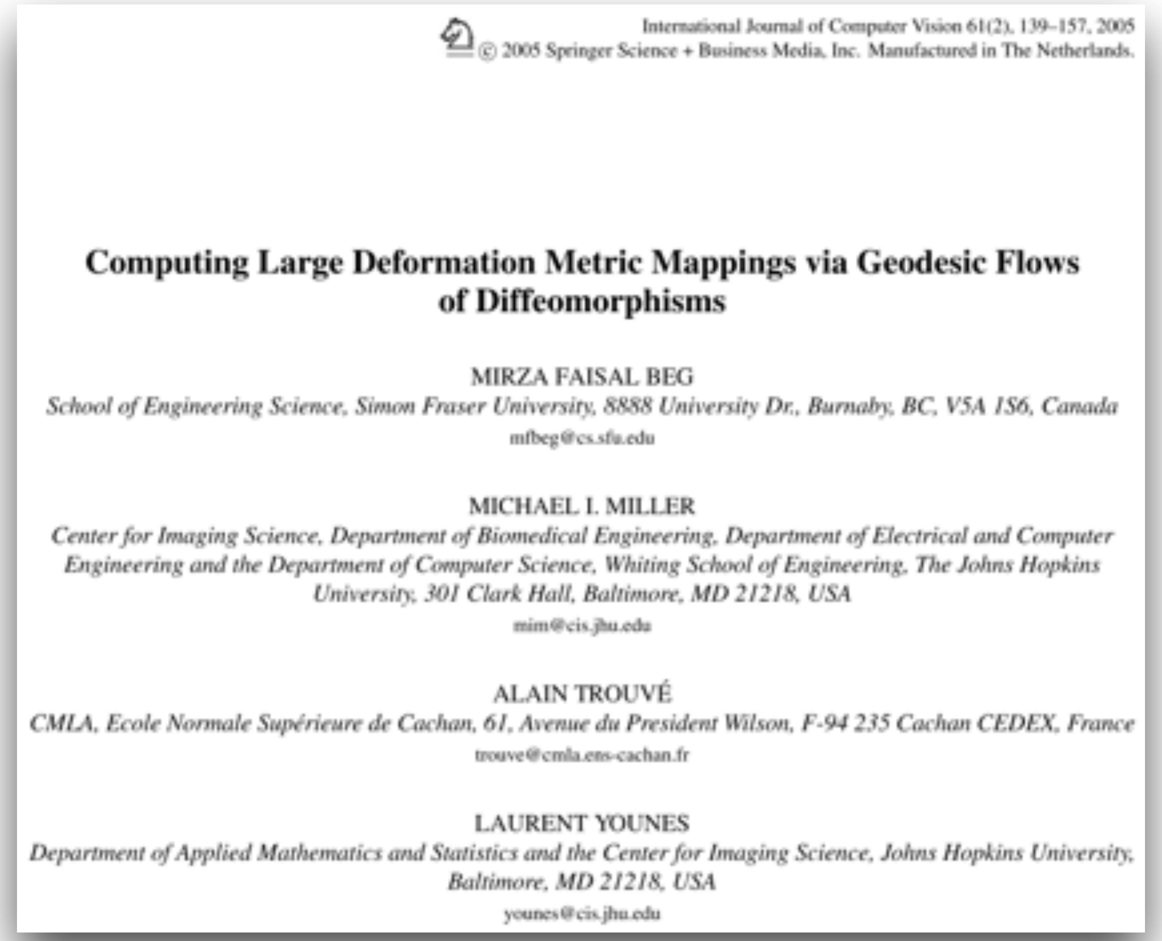
$$\frac{d\varphi}{dt} = \mathbf{v}(p, t)$$

Non-stationary ODE

$$\mathrm{Reg}(\mathbf{v}) = \sqrt{\int_0^1 ||L\mathbf{v}(t)||^2_{L^2} dt}$$

The update is additive, similar to the Classic Demon

$$\mathbf{v}_{j+1} = \mathbf{v}_j + \delta\mathbf{v}$$



International Journal of Computer Vision 61(2), 139–157, 2005

Computing Large Deformation Metric Mappings via Geodesic Flows of Diffeomorphisms

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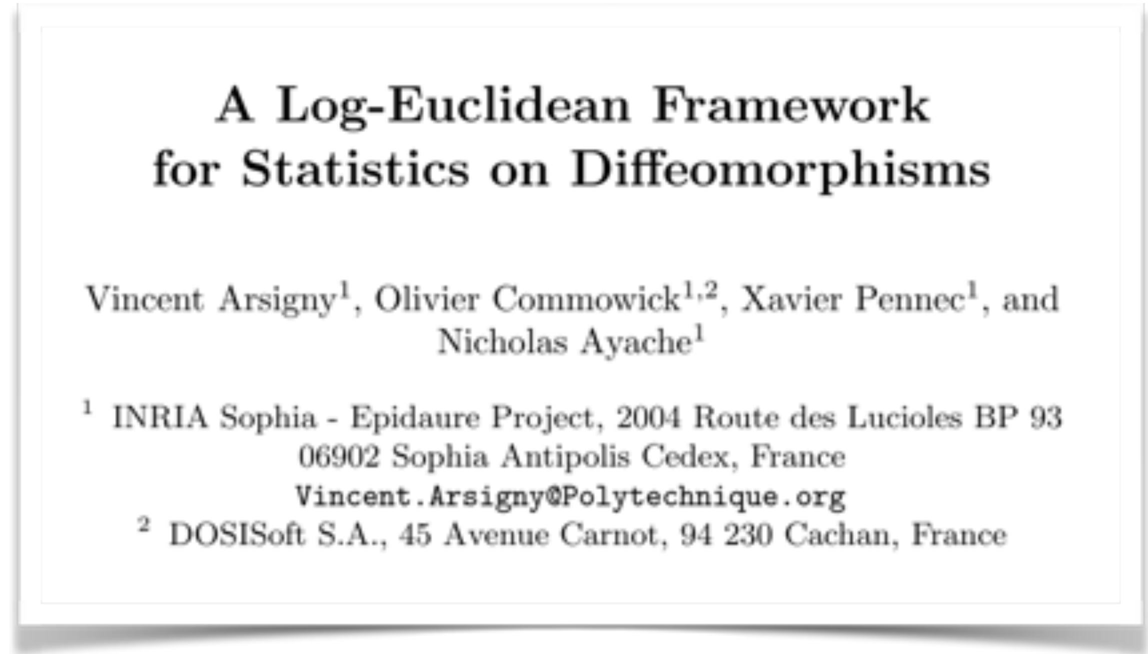
Kernel of the Regularization

$$\frac{d\varphi}{dt} = \mathbf{v}(p,t) \longrightarrow \frac{d\varphi}{dt} = \mathbf{v}(p)$$

From stationary to non-stationary ODE


A Log-Euclidean Framework for Statistics on Diffeomorphisms

Vincent Arsigny[1], Olivier Commowick[1,2], Xavier Pennec[1], and Nicholas Ayache[1]

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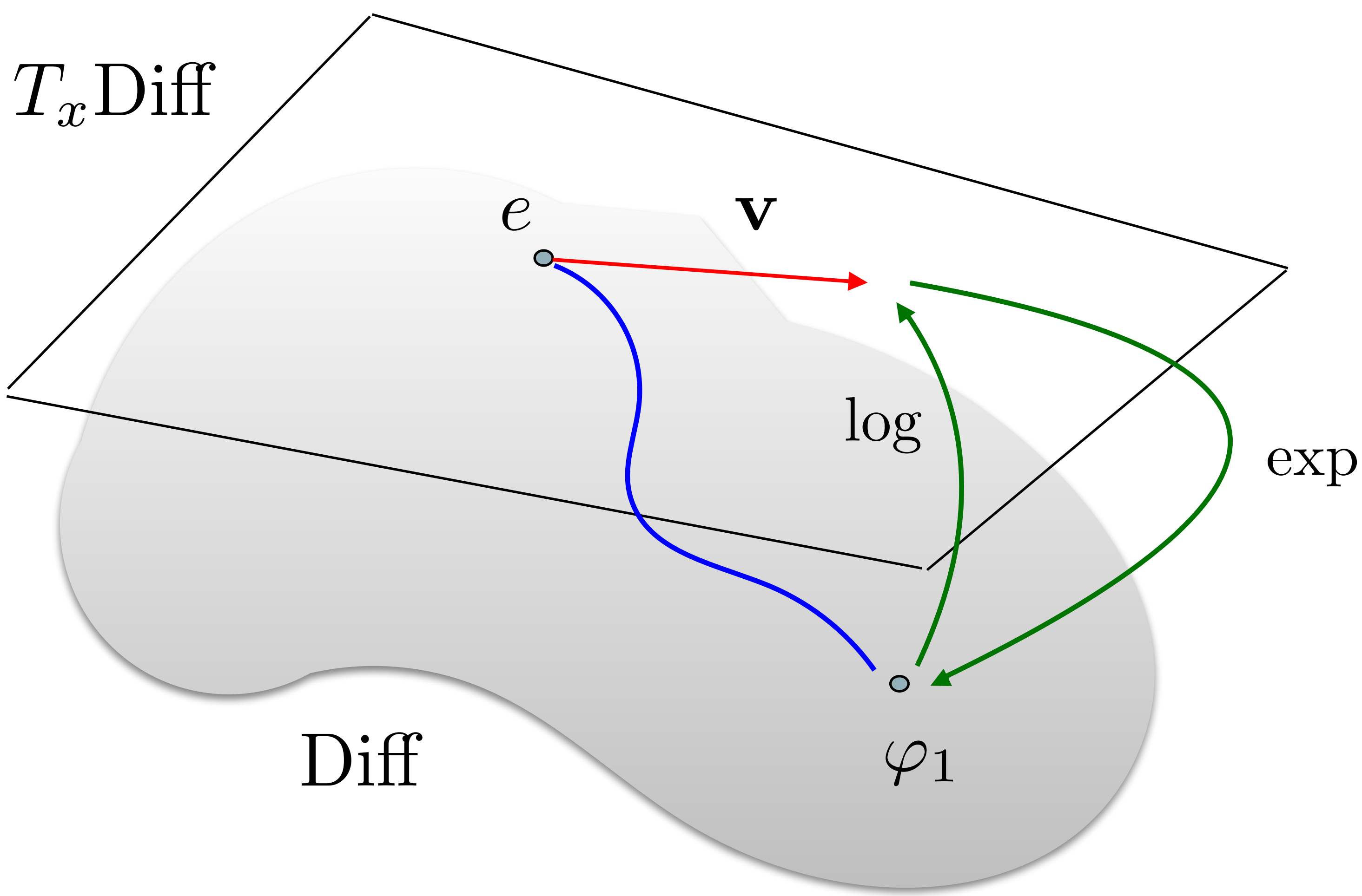

$$\log \rightarrow ISS$$
$$\exp \rightarrow SS$$

Stationary LDDMM

Registration of anatomical images using geodesic paths of diffeomorphisms parameterized with stationary vector fields

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$$\mathcal{E}(F, M, \mathbf{v}) = \frac{1}{\sigma_s}\mathrm{Sim}(F, M, \mathbf{v}) + \mathrm{Reg}(\mathbf{v})$$

LDDMM

Stationary LDDMM

$$\mathrm{Reg}(\mathbf{v}) = \sqrt{\int_0^1 ||L\mathbf{v}(t)||^2_{L^2}\, dt} \;\boxed{= ||L\mathbf{v}||_{L^2}}$$

Non-parametric Diffeomorphic Image Registration with the Demons Algorithm

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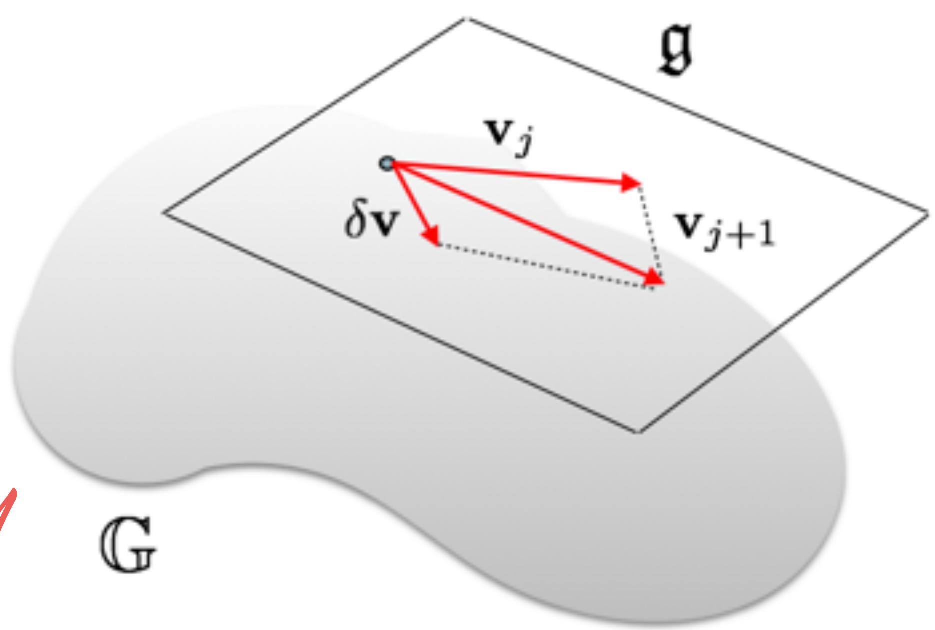
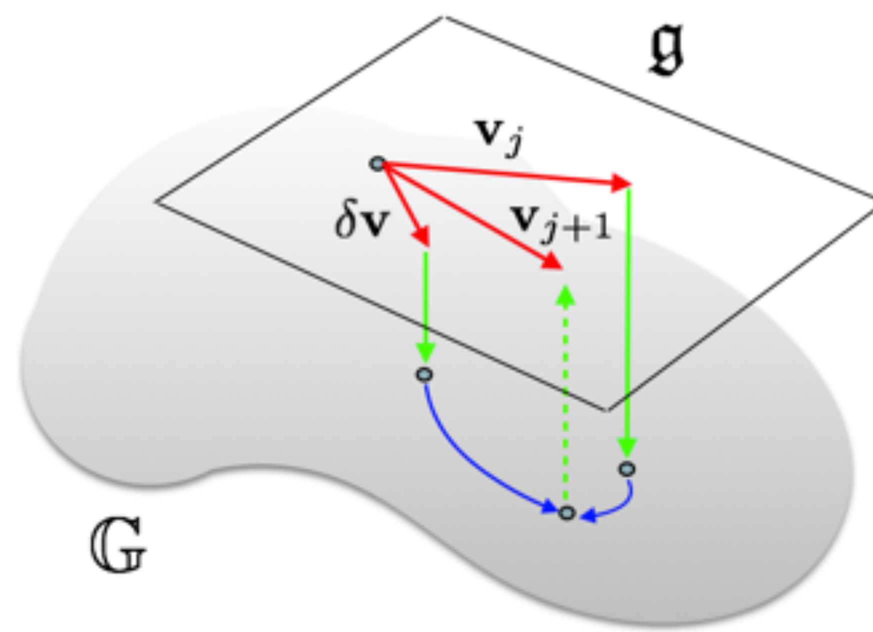
$$\mathcal{E}(F, M, \mathbf{v}) = \frac{1}{\sigma_s}\mathrm{Sim}(F, M, \mathbf{v}) + \mathrm{Reg}(\mathbf{v})$$





LDDMM
and
classical Demons

$$\mathbf{v}_{j+1} = \mathbf{v}_j + \delta\mathbf{v}$$

Diffeomorphic
Demons

$$\mathbf{v}_{j+1} = \log(\exp(\mathbf{v}_j) \circ \exp(\delta\mathbf{v}))$$

"Someone told me that each equation I included in the book would halve the sales."

- Stephen Hawking
"Short history of time"

Percentage of loss, if these slides were sold:

$$(1 - \frac{1}{2^6})100 = 98.4375$$

$$\mathcal{E}(F, M, \mathbf{v}) = \frac{1}{\sigma_s}\mathrm{Sim}(F, M, \mathbf{v}) + \mathrm{Reg}(\mathbf{v})$$

$$\mathrm{Sim}(F, M, \mathbf{v}) = ||M \circ \varphi - F||^2_{L^2}$$

$$\frac{d\varphi}{dt} = \mathbf{v}(p, t) \longrightarrow \frac{d\varphi}{dt} = \mathbf{v}(p)$$

$$\mathrm{Reg}(\mathbf{v}) = \sqrt{\int_0^1 ||L\mathbf{v}(t)||^2_{L^2} dt} = ||L\mathbf{v}||_{L^2}$$

$$\mathbf{v}_{j+1} = \mathbf{v}_j + \delta\mathbf{v}$$

$$\mathbf{v}_{j+1} = \log(\exp(\mathbf{v}_j) \circ \exp(\delta\mathbf{v}))$$

Stationary LDDMM

2007

Log-Demons

IC

Inverse Consistent LDDMM

Similarity function is "symmetrified"

$$\mathrm{Sim}(F, M, \mathbf{v}) = ||M \circ \varphi - F||^2_{L^2} + ||M - F \circ \varphi^{-1}||^2_{L^2}$$

SG

Symmetric Gradient optimisation LDDMM

The gradient in the optimisation scheme is computed as

$$\nabla F \circ \varphi^{-1} + \nabla M$$

Log

Log-Demons

Symmetric optimisation scheme

$$\mathcal{E}(F, M, \mathbf{v}, \mathbf{c}) = \mathrm{Sim}(F, M, \mathbf{c}) + \mathrm{dist}(\mathbf{c}, \mathbf{v}) + \mathrm{Reg}(\mathbf{v})$$

Dataset:

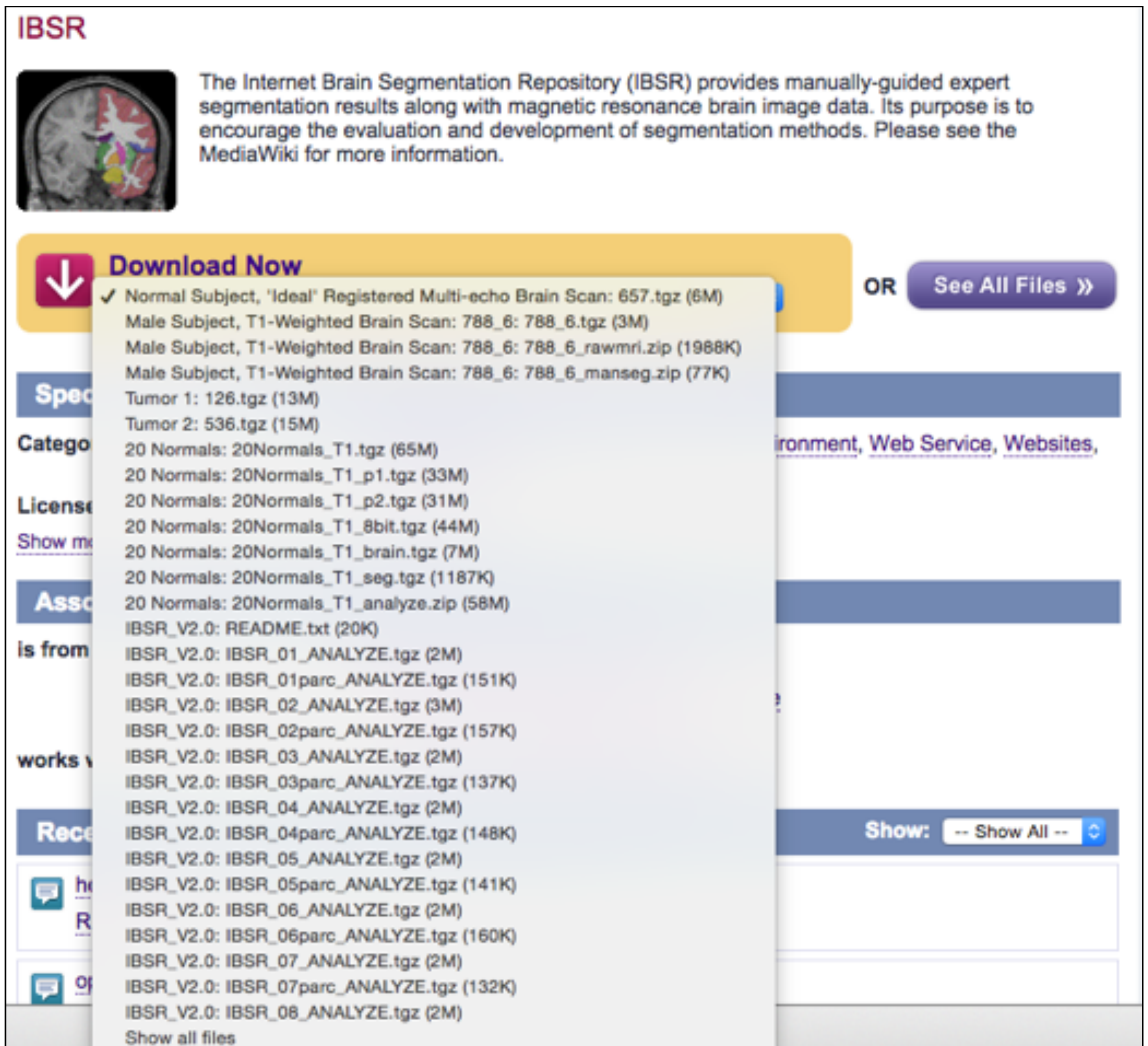
18 T1-MRI

from

Internet brain segmentation repository

Image size: 256 x 256 x 128

Voxel size: 0.94 x 0.94 x 1.50



1 of the images was randomly selected as template.

17 remaining where registered to this template.

Algorithm stops if:

- 100 iterations
- Magnitude of the update is negligible

Inverse consistent LDDMM. $RSSD = \frac{1}{2}\frac{\|I_0 \circ \varphi - I_1\|_2^2 + \|I_1 \circ \varphi^{-1} - I_0\|_2^2}{\|I_0 - I_1\|_2^2}$.

IC

$1/\sigma^2_{\text{sim}}$ \ α	1.0	0.01	0.0050	0.0025	0.0010	0.0001
1.0e3	91.56 ± 3.04	30.53 ± 3.76	21.51 ± 2.37	17.42 ± 4.16	*12.18 ± 3.17*	100.00 ± 0.00
	0.60 ± 0.24	0.44 ± 0.11	0.27 ± 0.07	0.17 ± 0.06	*-0.17 ± 0.64*	1.00 ± 0.00
1.0e4	90.97 ± 3.11	24.70 ± 3.10	17.55 ± 2.10	**13.88 ± 4.13**	*9.72 ± 3.72*	100.00 ± 0.00
	0.59 ± 0.24	0.31 ± 0.15	0.19 ± 0.08	**0.10 ± 0.05**	*-3.97 ± 12.28*	1.00 ± 0.00
1.0e5	90.97 ± 3.11	24.70 ± 3.10	17.55 ± 2.10	13.82 ± 4.00	*9.61 ± 3.57*	100.00 ± 0.00
	0.59 ± 0.24	0.31 ± 0.15	0.19 ± 0.08	0.10 ± 0.05	*-3.99 ± 12.28*	1.00 ± 0.00

Symmetric gradient LDDMM. $RSSD = \frac{\|I_0 \circ \varphi - I_1\|_2^2}{\|I_0 - I_1\|_2^2}$.

SG

$1/\sigma^2_{\text{sim}}$ \ α	1.0	0.01	0.0050	0.0025	0.0010	0.0001
1.0e3	91.66 ± 2.88	30.44 ± 3.44	22.02 ± 2.33	15.79 ± 1.68	*10.88 ± 1.21*	100.00 ± 0.00
	0.65 ± 0.22	0.44 ± 0.09	0.26 ± 0.08	0.11 ± 0.07	*0.02 ± 0.02*	1.00 ± 0.00
1.0e4	91.11 ± 2.73	28.61 ± 3.44	20.99 ± 2.38	**14.81 ± 1.59**	*10.09 ± 1.35*	100.00 ± 0.00
	0.63 ± 0.23	0.39 ± 0.12	0.23 ± 0.10	**0.10 ± 0.07**	*0.01 ± 0.01*	1.00 ± 0.00
1.0e5	91.11 ± 2.73	28.83 ± 3.62	21.49 ± 2.46	15.38 ± 2.74	*10.09 ± 1.34*	100.00 ± 0.00
	0.63 ± 0.23	0.39 ± 0.12	0.25 ± 0.10	0.11 ± 0.07	*0.01 ± 0.01*	1.00 ± 0.00

$$\mathcal{E}(F, M, \mathbf{v}) = \frac{1}{\sigma_s}\text{Sim}(F, M, \mathbf{v}) + \text{Reg}(\mathbf{v})$$

$$\text{Reg}(F, M, \mathbf{v}) = ||L\mathbf{v}||^2_{L^2} = ||\alpha\mathbf{v} + \gamma||^2_{L^2}$$

Demons: parameters selection

Diffeomorphic Demons. $RSSD = \frac{\| I_0 \circ \varphi - I_1 \|_2^2}{\| I_0 - I_1 \|_2^2}$.

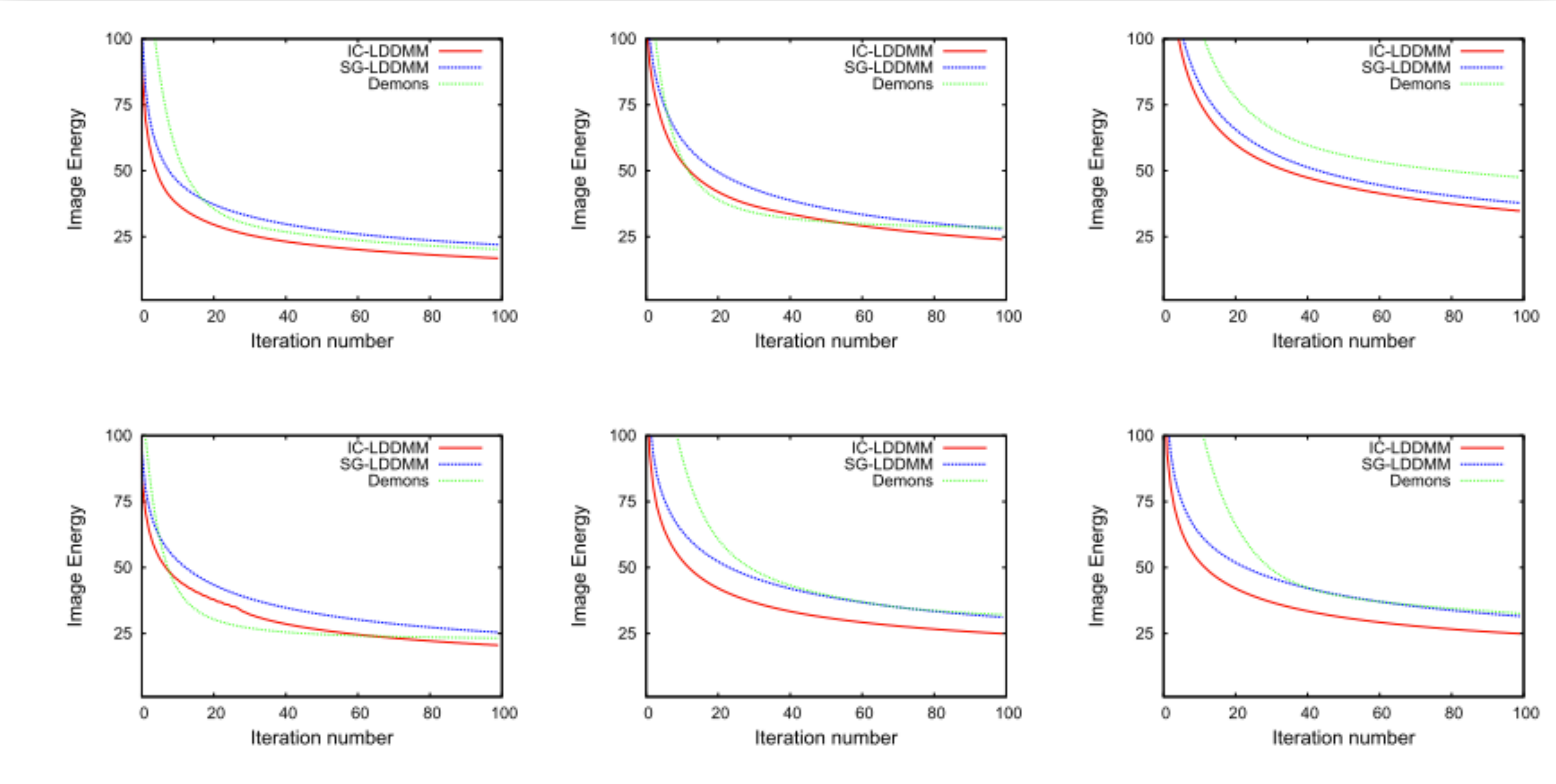
τ \ σ_u	1.0	1.5	2.0	2.5	3.0
0.5	**14.88 ± 1.74** **0.07 ± 0.03**	20.91 ± 2.50 0.15 ± 0.04	31.58 ± 3.67 0.31 ± 0.05	40.71 ± 4.34 0.47 ± 0.06	48.15 ± 4.61 0.59 ± 0.06
1.0	*9.95 ± 1.18* *0.00 ± 0.00*	*13.99 ± 1.69* *0.02 ± 0.02*	21.73 ± 2.73 0.11 ± 0.05	29.37 ± 3.72 0.23 ± 0.07	36.19 ± 4.47 0.35 ± 0.09
2.0	*10.91 ± 1.79* *-0.01 ± 0.02*	*12.60 ± 1.26* *-0.00 ± 0.01*	*18.06 ± 2.15* *0.02 ± 0.02*	24.51 ± 3.01 0.10 ± 0.04	30.59 ± 3.75 0.19 ± 0.07

Log

Results

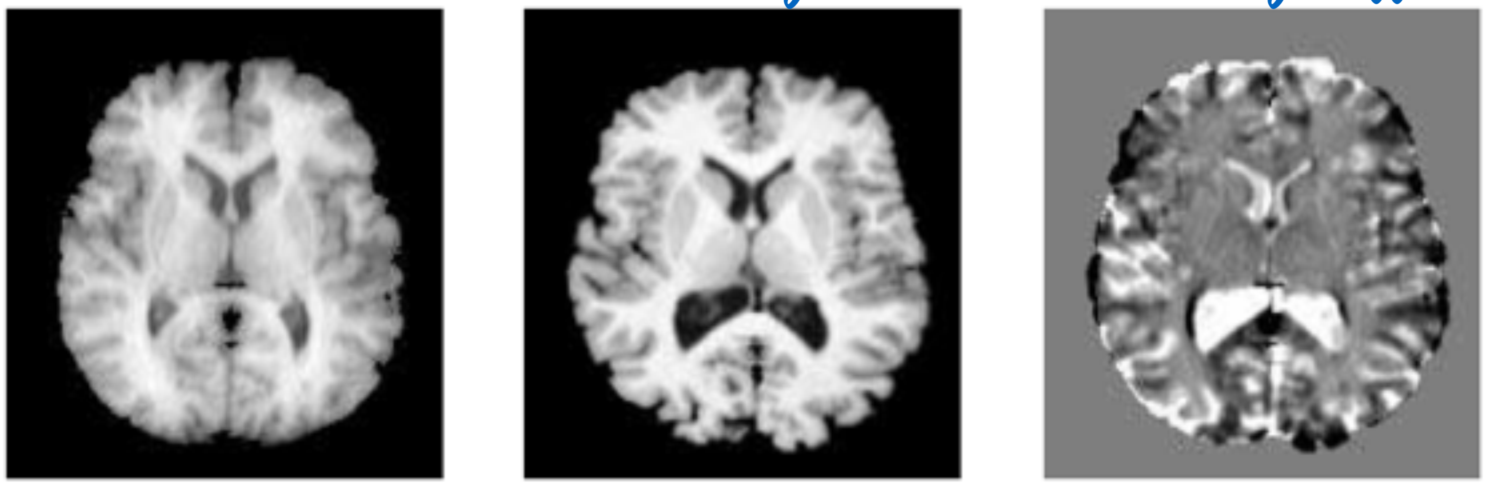
	$RSSD_{(I_0 \circ \varphi, I_1)}$ (%)	$RSSD_{(I_1 \circ \varphi^{-1}, I_0)}$ (%)	$\|\cdot\|_V^2$	$\|\cdot\|_{fro}^2$	? Approx. time (per iteration)
IC-LDDMM	13.42 ± 4.23	14.43 ± 4.24	140.52 ± 28.84	0.17 ± 0.05	90.64
SG-LDDMM	14.81 ± 1.59	15.47 ± 2.50	166.62 ± 12.05	0.20 ± 0.03	53.23
Demons	14.88 ± 1.74	19.00 ± 4.72	2626.70 ± 6069.90	0.13 ± 0.01	41.54

?where does alpha came from?

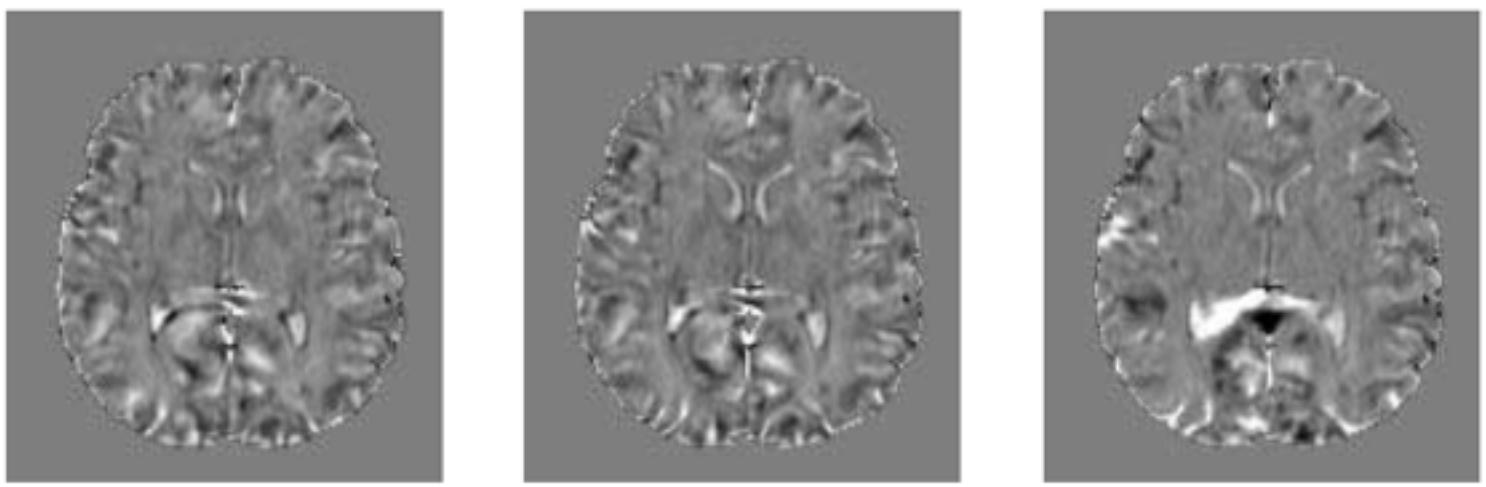


Visual Assessment

Fixed *Moving* *Fixed-Moving difference*

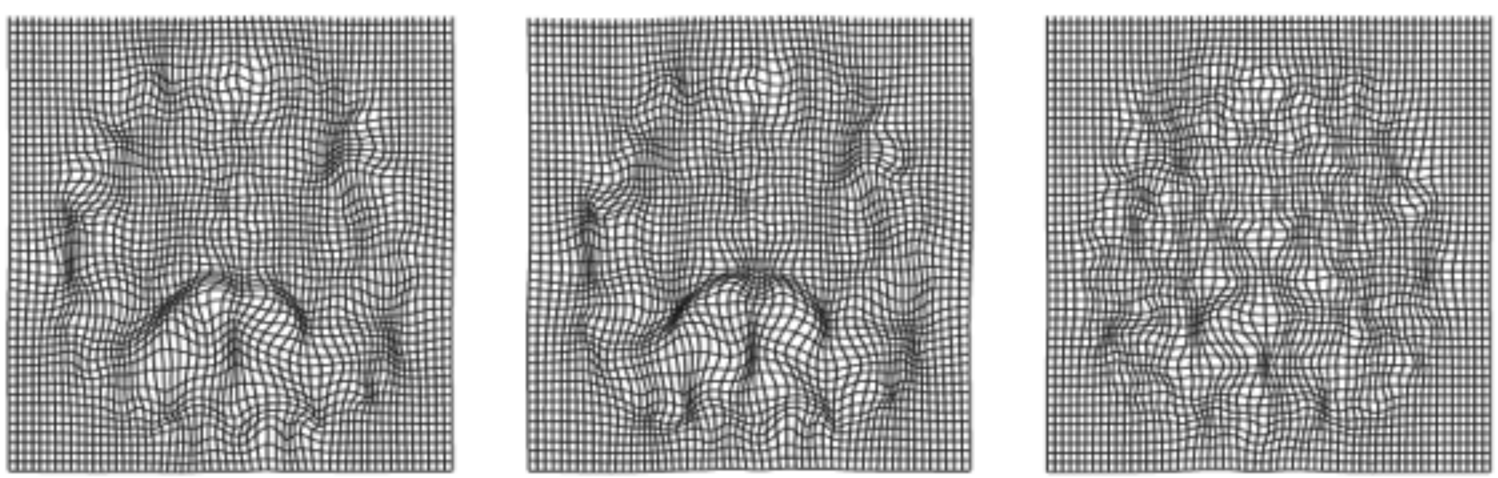


Images difference after registration



IC LDDMM *SG LDDMM* *Demons*

Corresponding Diffeomorphisms



SG-LDDMM IC-LDDMM vs Demons: the conclusion

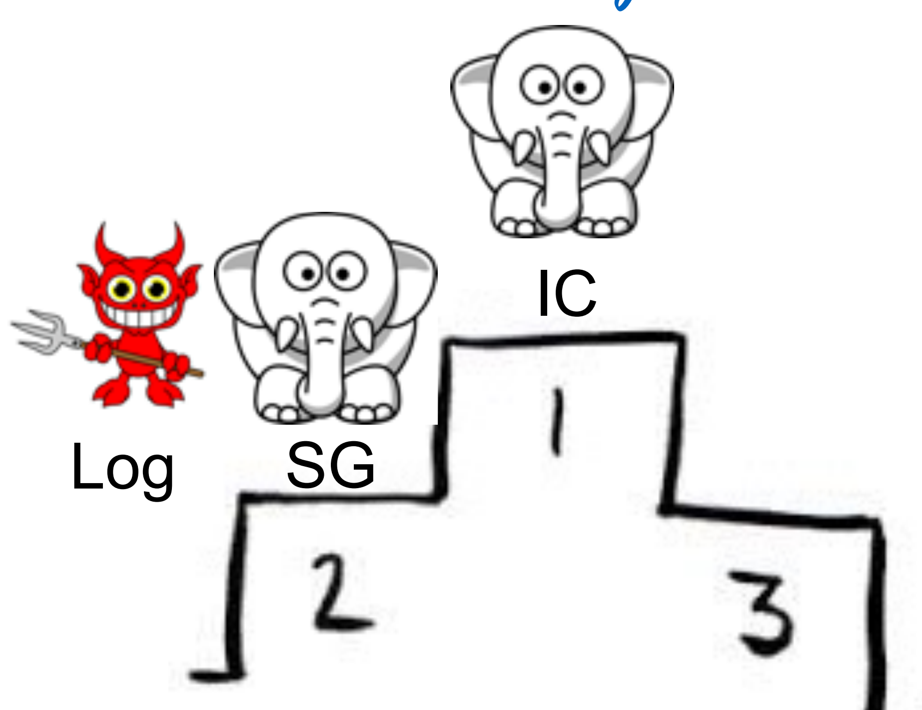
All of the Algorithms are strongly influenced by the choice of the parameters.
(Some of them may provide non-diffeomorphic transformations)!

Both algorithms provided similar RSSD (between 13% and 15%).
IC provides the best results in most cases

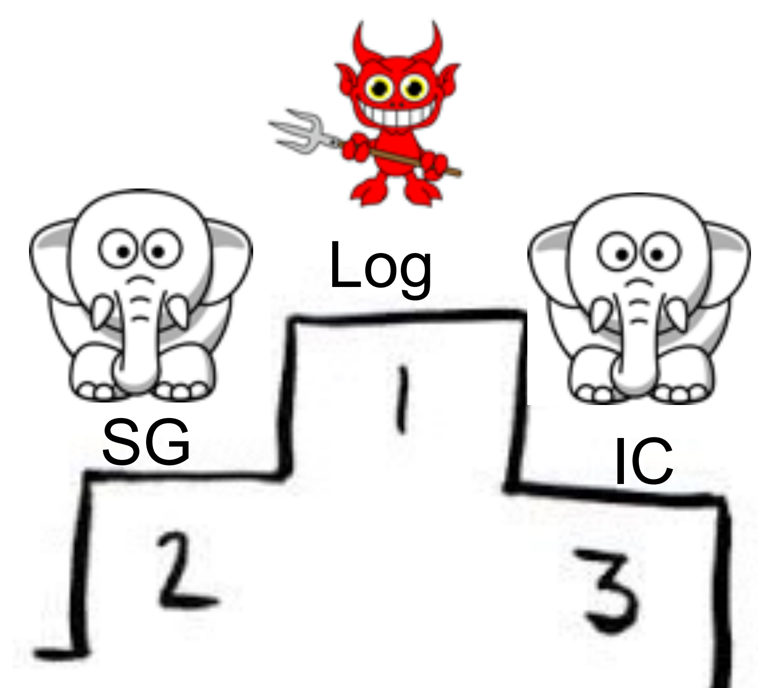
Log-Demons is faster than Stationary LDDMM, both SG and IC.

> In conclusion, both methods may be considered close from a theoretical point of view and equivalent from a practical point of view for registration purposes. Diffeomorphic Demons demonstrated similar intensity matching performances to stationary LDDMM at a slightly lower computational cost.

Accuracy



Computational time



Limitations and Contributions:

MICCAI workshop

Nothing new has been invented…

Instead of trying to invent something new, authors sit down and analyses what has been done so far!

18 citations

Authors of this comparison are the authors of one of the two methods compared…

Laconic bibliography…

Missing computational time analysis.

Norms of the vector fields are not much investigated.

Only 1 measure of similarity is taken into account.

Two methods are explained in details, with pros, cons, and, with examples!

Laconic bibliography!
No more than what you may need!

Great introduction!

Good paper to learn about diffeomorphic image registration!

Thanks for your attention!

Questions?